NASA TECHNICAL TRANSLATION NASA TT F-12,966

THE PROBLEM OF OSCILLATIONS OF A THIN PROFILE IN A SUBSONIC FLOW NEAR A RIGID BOUNDARY

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Translation of "K Zadache O Kolebaniyakh Tonkogo Profilya V Dozvukovom Potoke Vblizi Tverdoy Granitsy", Prikladnaya Mekhanika, Vol. 5, No. 7, 1969, pp. 119-121.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 MAY 1970

THE PROBLEM OF OSCILLATIONS OF A THIN PROFILE IN A SUBSONIC FLOW NEAR A RIGID BOUNDARY

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Reference [2] obtained the integral equation

$$\int_{-1}^{+1} \gamma(s) K(x-s)\,ds = V_y(x) \quad (-1 \leqslant x \leqslant +1). \tag{1}$$

for the complex amplitude of the pressure drop along a thin profile

$$\gamma = \frac{p_- - p_+}{\varrho V_0^2}$$

Here

$$\begin{gathered} K(z) = -\frac{\sqrt{1-M^2}}{2\pi}\exp(-ikz)\int_{-\infty}^{z}\exp\frac{ik\xi}{1-M^2}\,\frac{\pi}{2i}\left\{H_1^{(2)}\left(\frac{kM}{1-M^2}\cdot\frac{1}{|\xi|}\right.\right. - \\ -\frac{1}{2}\left[H_0^{(2)}\left(\frac{kM}{1-M^2}\sqrt{A}\right)-H_2^{(2)}\left(\frac{kM}{1-M^2}\sqrt{A}\right)\right]\frac{k^2M^2}{1-M^2}\cdot\frac{4h}{\sqrt{A}} - \\ \left.-H_1^{(2)}\left(\frac{kM}{1-M^2}\sqrt{A}\right)\frac{kM}{1-M^2}\,\frac{\xi^2-16(1-M^2)h^2}{A^{3/2}}\right\}d\xi; \\ A = \xi^2 + 16(1-M^2)h^2; \end{gathered} \tag{2}$$

x and s are the coordinate and the arc abscissa along the profile in the direction of the forward flow, equated to the half-chord; h is the distance of the profile from the rigid boundary, measured in chord fractions; $k = \omega/V_0$ is the Strouhal number; M is the Mach number for the undisturbed flow; $H_n^{(2)}$ is a Hankel function of the second type; V_y is the complex amplitude of the

* Numbers in the right margin indicate pagination in the foreign text.

vertical velocity along the profile.

Equations (1), with kernel (2), is the generalization of the known Possio equation cited in [5] for the case of motion near a rigid boundary.

The kernel of equation (1) has the peculiarities of a type of first order pole and of the logarithm when x = 3. The solution of $\gamma(x)$ has the peculiarity of the type $\delta^{-1/2}$, where δ is the distance from the leading edge, x = -1. The function $\gamma(x)$ should be bounded at the trailing edge.

The collocation method was used in [3, 4] to solve equations of the type at (1), and this requires calculation of certain special integrals. The discrete vortices method [1] is quite effective for solving analogous problems in the case of an incompressible fluid, because the integral equation can be introduced in the system of algebraic equations, the coefficients of which are values of the kernel of the integral equation. Let us use this method to solve equation (1), and in this case, it can be called the discrete dipole /120 method. The sought for pressure drop is a function proportional to the function of the distribution of the dipoles for the acceleration potential along the profile [5].

Let us replace the continuous distribution by discrete distribution, so we can replace the integral in the relationship at (1) with a quadrature formula with nodes at certain points $S_j = -1 + (4j - 3/4N)$. We will require that the closure resulting from the replacement of the integral by a finite sum vanish at some other points $x_i = -1 + (4i - 1/4N)$, where N is the number of discrete dipoles. We thus arrive at a system of algebraic equations

$$\begin{aligned} &\frac{2}{N}\sum_{j=1}^{N}\gamma_j^{(1)}K^{(1)}(x_i - s_j) - \frac{2}{N}\sum_{j=1}^{N}\gamma_j^{(2)}K^{(2)}(x_i - s_j) = V_y^{(1)}; \\ &\frac{2}{N}\sum_{j=1}^{N}\gamma_j^{(1)}K^{(2)}(x_i - s_j) + \frac{2}{N}\sum_{j=1}^{N}\gamma_j^{(2)}K^{(1)}(x_i - s_j) = V_y^{(2)}, \end{aligned} \tag{3}$$

where the upper index (1) denotes the real part of the amplitude of the pressure drop along the profile, the kernel of equation (1), and the vertical profile velocity.

Since nowhere is $x_i \neq S_j$, the coefficients of the system at (3) are regular. Questions of existence, of uniqueness of the solution, and of error in the calculations are not considered here.

The lift and moment coefficients relative to the center of the profile can be calculated by using the approximate formulas

$$C_y = \frac{2}{N} \sum_{j=1}^{N} \gamma_j; \quad C_m = \frac{1}{N} \sum_{j=1}^{N} s_j \gamma_j. \tag{4}$$

Figures 1 through 4 show the results of the calculations for the unsteady aerodynamic characteristics for a thin profile for various Mach and Strouhal numbers, and distances from the rigid boundary.

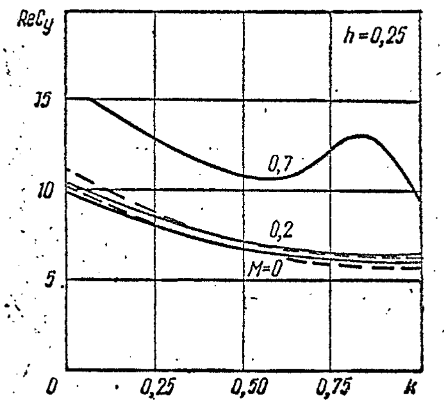

Figure 1.

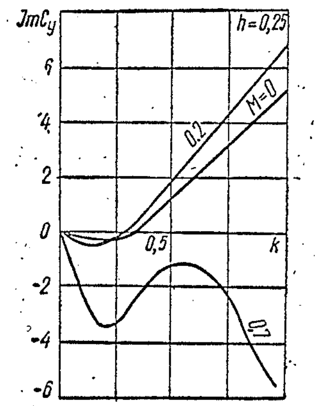

Figure 2.

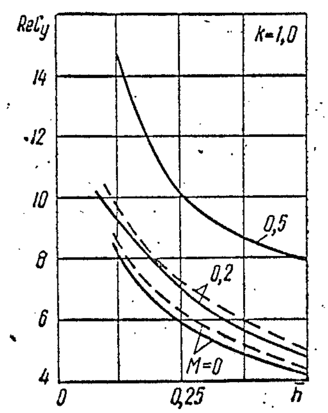

Figure 3.

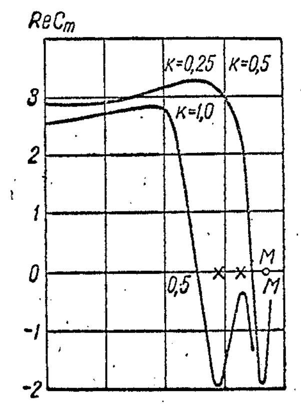

Figure 4. /121

As will be seen, resonance phenomena occur when a compressed subsonic flow takes place over a thin profile located near a rigid boundary. V. B. Kurzin [3] was first to point out these phenomena in connection with the problem of oscillations of lattices in subsonic flows. Similar results were obtained in the problem of oscillations of multiple surfaces [4]. The results obtained by V. B. Kurzin, deduced from the solution of equation (1) by the collocation method, are plotted in the figures by the dashed lines for purposes of comparison.

The crosses in Figure 4 mark the resonant values of the numbers for the Strouhal number $k = 1.0$, and the points those for $k = 0.25$ when $h = 0.5$.

Analysis of the results obtained reveals that at high subsonic velocities, or at high frequencies, the compressibility of the fluid has a significant effect on the flow over profiles near a rigid screen.

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Submitted
18 December 1967

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Translated for the National Aeronautics and Space Administration under contract No. NASw-2038 by Translation Consultants, Ltd., 944 South Wakefield Street, Arlington, Virginia 22204.